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Equilibrium Near and Far Electromagnetic Field Structures at a Flat Boundary of the Half-Space Filled with a Homogeneous Dielectric (Magnetic) Medium

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Abstract—The most general expression for at a flat interface between a semiinfinite homogeneous dielectric (magnetic) medium and vacuum is obtained. The medium is characterized by the frequency-dependent dielectric permittivity and magnetic permeability.

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The density of electromagnetic field modes (or the density of states, DOS) at the surface of a condensed medium differs from that in the far-field zone, that is, at distances much greater than the characteristic radiation wavelength. This is also valid for the electromagnetic field structure in meso- and nanostructural media. Changes in the local DOS (LDOS) influence the probabilities of elementary quantum processes related to the photon emission. These processes involve variations in the spectra, transition probabilities, intensities of the spontaneous (nonequilibrium) and equilibrium thermal radiation, and the Raman and Rayleigh scattering intensities [1–6].

In the case of equilibrium electromagnetic radiation in vacuum, the DOS is independent of the coordinates and can be described in terms of the classical formula: $D(\omega) = \omega^2/\pi^2 c^3$. In the general case of a spatially inhomogeneous medium, the DOS depends on the coordinates (radius vector $\mathbf{r}$) and is determined using the relation [7]

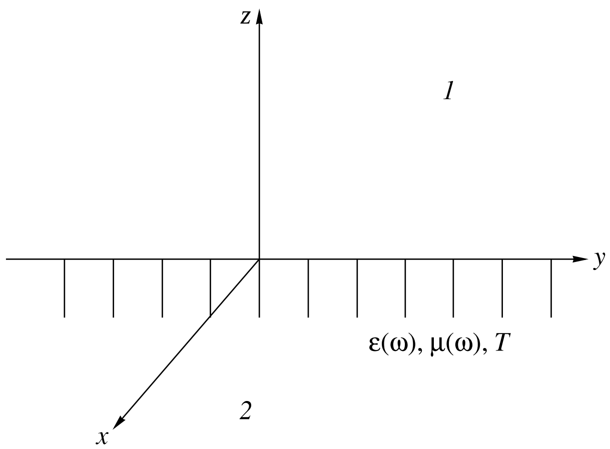
$$U(\mathbf{r}, \omega) = D(\mathbf{r}, \omega)\frac{\hbar\omega}{\exp(\hbar\omega/k_B T) - 1}, \quad (1)$$

where $U(\mathbf{r}, \omega)$ is the spectral density of the electromagnetic field energy (per unit volume), $\hbar$ is the Planck constant, k_B is the Boltzmann constant, and T is the absolute temperature of the medium.

In accordance with definition (1), the LDOS can be calculated provided that the $U(\mathbf{r}, \omega)$ function is known. However, the determination of $D(\mathbf{r}, \omega)$ in each particular case is a quite complicated task: even calculations for a flat interface between a semiinfinite, homogeneous and isotropic polarizable medium encounters certain difficulties. Recently, Dorofeyev *et al.* [8] calculated the LDOS inside a flat vacuum gap between two half-spaces having different temperatures and magnetic permeabilities. The calculation was based on the generalized Kirchhoff law. A particular case of the interface between a semiinfinite medium and vacuum follows from the general result [8] as the gap width tends to the infinity. More recently, Joulain *et al.* [9] calculated the LDOS for a semiinfinite medium–vacuum interface using an imaginary part of the trace of the electromagnetic field dyad. Since the expressions obtained in [8, 9] are difficult to compare in detail, the question concerning a correct representation of the $D(\mathbf{r}, \omega)$ function for the system under consideration still remains open.

In this context, it was of interest to solve the problem using an alternative method that has been developed in our recent investigations of fluctuational electromagnetic forces [10, 11]. This Letter presents the results of such analysis and demonstrates that the proposed method provides for a considerable generality of the obtained solutions and can be successfully used for solving a broad class of problems related to the static and dynamic effects of fluctuational electromagnetic interactions [10–12]. A final expression for the DOS obtained (for $\mu(\omega) = 1$) using the proposed method fully coincides with the results obtained in [9], where a nonmagnetic medium was studied.

Let us consider a flat interface between vacuum (upper half-space) and a homogeneous isotropic medium (lower half-space) occurring at an absolute temperature T. The medium (see figure) is characterized by the permittivity $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$. The energy density (per unit volume) of the fluc-



Schematic diagram of the system under consideration: (*1*) vacuum ($z > 0$); (*2*) interface (*xy* surface).

tuational electromagnetic field at a distance z from the interface can be written as

$$U(z) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\int_{-\infty}^{\infty}\frac{dk_y}{2\pi}\left[\frac{(\mathbf{E}^2)_{\omega\mathbf{k}} + (\mathbf{B}^2)_{\omega\mathbf{k}}}{8\pi}\right], \quad (2)$$

where $(\mathbf{E}^2)_{\omega\mathbf{k}}$ and $(\mathbf{B}^2)_{\omega\mathbf{k}}$ are the spectral densities of the fluctuational electric and magnetic field components corresponding to the field operators $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, respectively. These spectral densities can be expressed in terms of the retarded Green's function components for a photon in the medium [13]:

$$\begin{aligned}(E_i(z)E_j(z'))_{\omega k} &= \frac{i}{2}\coth\frac{\omega\hbar}{2k_{\mathrm{B}}T}\frac{\omega^2}{c^2} \\ &\times[D_{ij}(\omega\mathbf{k}, z, z') - D^*_{ji}(\omega\mathbf{k}, z, z')] \\ &(i, j = x, y, z),\end{aligned} \quad (3)$$

$$\begin{aligned}(B_i(z)E_j(z'))_{\omega k} &= \frac{i}{2}\coth\frac{\omega\hbar}{2k_{\mathrm{B}}T}\frac{\omega^2}{c^2} \\ \times\,\mathrm{curl}_{il}\mathrm{curl}'_{jm}&[D_{lm}(\omega\mathbf{k}, z, z') - D^*_{ml}(\omega\mathbf{k}, z, z')],\end{aligned} \quad (4)$$

where

$$\mathrm{curl}_{il} = e_{ipl}\frac{\partial}{\partial x_p}, \quad \mathrm{curl}'_{jm} = e_{jqm}\frac{\partial}{\partial x'_q},$$

$$\frac{\partial}{\partial(x, y)} = i(k_x, k_y),$$

$$\frac{\partial}{\partial(x', y')} = -i(k_x, k_y), \quad \frac{\partial}{\partial(z, z')} = -q_0,$$

$$q_0 = (k^2 - \omega^2/c^2)^{1/2}.$$

In the above formulas, e_{ipl} is the Levi–Civita antisymmetric tensor, c is the light velocity in vacuum, $k^2 = k_x^2 + k_y^2$, and (k_x, k_y) is a two-dimensional wavevector. The Green's function components can be determined using the following system of equations [13]:

$$\begin{aligned}&\left(\mathrm{curl}_{im}\mathrm{curl}_{ml} - \frac{\omega^2}{c^2}\varepsilon(\omega)\delta_{il}\right)D_{ik}(\omega, \mathbf{r}, \mathbf{r}') \\ &= -4\pi\hbar\mu(\omega)\delta_{ik}\delta(\mathbf{r} - \mathbf{r}'),\end{aligned} \quad (5)$$

$$\begin{aligned}D_{ik}(\omega, \mathbf{r}, \mathbf{r}') &= \int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\int_{-\infty}^{\infty}\frac{dk_y}{2\pi}D_{ik}(\omega\mathbf{k}, z, z') \\ &\times\exp[i(k_x(x - x') + k_y(y - y'))].\end{aligned} \quad (6)$$

Solving Eqs. (5) with the corresponding boundary conditions at $z = 0$, and taking into account relations (3) and (4), we determine the spectral densities of the component fields as

$$\begin{aligned}(\mathbf{E}^2)_{\omega\mathbf{k}} &= \frac{4\pi\omega^2}{c^2}\hbar\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\mathrm{Im}\left(\frac{1}{q_0}\right) \\ &+ 2\pi\hbar\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\mathrm{Im}\left\{\frac{\exp(-2q_0z)}{q_0}\right. \\ &\left.\times[(2k^2 - \omega^2/c^2)\Delta_e(\omega) + (\omega^2/c^2)\Delta_m(\omega)]\right\},\end{aligned} \quad (7)$$

$$\begin{aligned}(\mathbf{B}^2)_{\omega\mathbf{k}} &= \frac{4\pi\omega^2}{c^2}\hbar\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\mathrm{Im}\left(\frac{1}{q_0}\right) \\ &+ 2\pi\hbar\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\mathrm{Im}\left\{\frac{\exp(-2q_0z)}{q_0}\right. \\ &\left.\times[(2k^2 - \omega^2/c^2)\Delta_e(\omega) + (\omega^2/c^2)\Delta_m(\omega)]\right\},\end{aligned} \quad (8)$$

where

$$\Delta_e(\omega) = \frac{q_0\varepsilon(\omega) - q}{q_0\varepsilon(\omega) + q}, \quad \Delta_m(\omega) = \frac{q_0\mu(\omega) - q}{q_0\mu(\omega) + q},$$

$$q = (k^2 - (\omega^2/c^2)\varepsilon(\omega)\mu(\omega))^{1/2}.$$

Using expressions (1), (7), and (8), we eventually obtain a formula for the spectral energy density that can be written (assuming that $\omega > 0$) as

$$\begin{aligned}U(z, \omega) &= 0.5(2\pi)^{-4}\int d^2k[(\mathbf{E}^2)_{\omega\mathbf{k}} + (\mathbf{B}^2)_{\omega\mathbf{k}}] \\ &= \rho_1(\omega) + \rho_2(z, \omega).\end{aligned} \quad (9)$$

Here, $\rho_1(\omega)$ is a function that is independent of the coordinate z and determined by the first terms in expressions (7) and (8). The calculation of $\rho_1(\omega)$ yields the spectral energy density of the equilibrium radiation at a temperature T (per unit volume of the medium), including the zero-point energy:

$$\rho_1(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}\left(\frac{1}{2} + \frac{1}{\exp(\hbar\omega/k_B T) - 1}\right). \qquad (10)$$

For $\rho_2(z, \omega)$, expressions (7)–(9) yield

$$\rho_2(z, \omega) = \frac{\hbar}{4\pi^2}\coth^2\left(\frac{\hbar\omega}{2k_B T}\right) \times \int_0^{\omega/c} dk k^3 \mathrm{Re}\left[\frac{\exp(2i|q_0|z)}{|q_0|}(\Delta_e(\omega) + \Delta_m(\omega))\right] + \frac{\hbar}{4\pi^2}\coth\left(\frac{\hbar\omega}{2k_B T}\right)\int_{\omega/c}^{\infty} dk k^3 \times \mathrm{Im}\left[\frac{\exp(-2q_0 z)}{q_0}(\Delta_e(\omega) + \Delta_m(\omega))\right]. \qquad (11)$$

Formulas (8)–(10) exactly coincide with the results obtained in [9] for $\mu(\omega) = 1$, if we take into account that $\Delta_e(\omega)$ and $\Delta_m(\omega)$ are equivalent to the Fresnel reflection coefficients R_s and R_p for the electromagnetic waves (corresponding to the field components with different polarizations in terms of [9]).

Apparently, for $z \longrightarrow \infty$, the dominating contribution to the DOS according to relation (9) is provided by the $\rho_1(\omega)$ term. However, the coordinate-dependent term $\rho_2(z, \omega)$ even at relatively large distances from the interface in the half-space $z > 0$ can provide a significant, spatially oscillating contribution to the DOS (due to the first integral in formula (11)). The range of distances for which the DOS tends to $D(\omega) = \omega^2/\pi^2 c^3$ can be estimated from the condition $\omega_0 z/c \approx 1$, where ω_0 is a characteristic frequency of the electromagnetic spectrum. Therefore, once the interface features a certain field mode, the oscillating contribution of this mode to the DOS will be manifested at distances on the order of the mode wavelength, while the particular dependence of the DOS on the distance from the interface will be determined by the form of the $\varepsilon(\mu)$ and $\mu(\omega)$ functions.

In the limit of the near-field zone, whereby $\omega_0 z/c \longrightarrow 0$, the main contribution to $\rho_2(z, \omega)$ is due to the second integral in formula (11). In this case, the DOS can be expressed as

$$D(z, \omega) = \frac{\hbar}{8\pi^2 z^3}\frac{1}{\omega} \times \left[\mathrm{Im}\left(\frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1}\right) + \mathrm{Im}\left(\frac{\mu(\omega) - 1}{\mu(\omega) + 1}\right)\right]. \qquad (12)$$

This formula predicts a resonance growth in the DOS for $\mathrm{Re}[\varepsilon(\omega)] = -1$ and $\mathrm{Re}[\mu(\omega)] = -1$, whereby each resonant mode with the frequency ω_0 is characterized by the corresponding effective range $z_0 = c/\omega_0$.

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